



MUTUAL

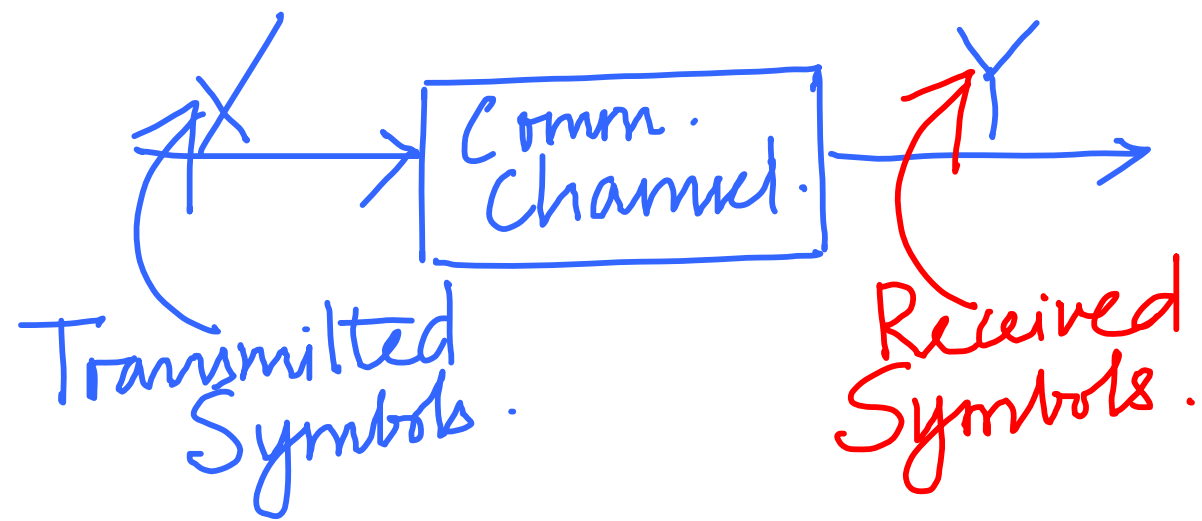
INFORMATION

$X, Y \leftarrow$ Two sources.

$$I(X; Y) = H(Y) - H(Y|X)$$

$$= H(X) - H(X|Y)$$

Mutual
Information between
 X, Y



$$I(X; Y) = H(X) - H(X|Y)$$

$$\begin{aligned}
 & H(X, Y) = H(Y) + H(X|Y) \\
 \Rightarrow & H(X|Y) = H(X, Y) - H(Y) \\
 & = H(X) - (H(X, Y) - H(Y))
 \end{aligned}$$

$$I(X;Y) = H(X) + H(Y) - H(X,Y)$$

$$H(X) - H(X|Y)$$

$$H(Y) - H(Y|X).$$

Transmitted.

X



Received.

Y

$$I(X;Y) = H(X) - H(X|Y)$$

Uncertainty
in X

remaining
uncertainty in
X on observing
Y.

uncertainty about
~~X~~ "resolved" on observing
Y.

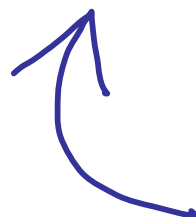
Plays a fundamental
role in characterizing
communication across
channel.

For a good channel, one would
expect $I(X; Y)$ to be
large.

$\Rightarrow$ on observing Y, able
to resolve ~~X~~ with a high
degree of reliability.

Properties of $I(X; Y)$:

$$I(X; Y) = I(Y; X)$$


$$\begin{aligned} &= H(X) - H(X|Y) \\ &= H(Y) - H(Y|X) \\ &= I(Y; X). \end{aligned}$$

$$I(X; Y) \geq 0$$

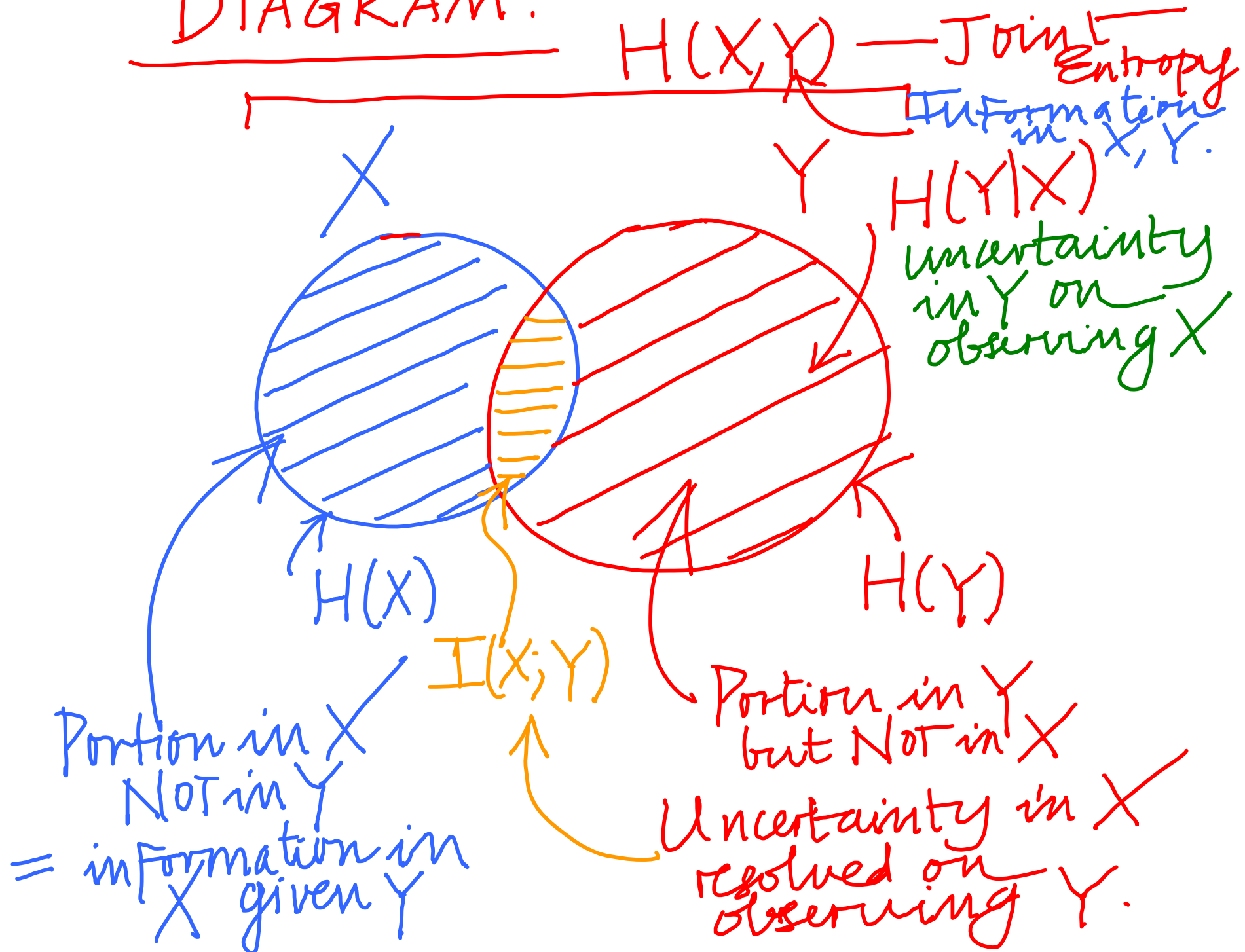
$$= H(X) + H(Y) - H(X, Y)$$

$$H(X) + H(Y) \geq H(X, Y)$$

$$\Rightarrow H(X) + H(Y) - H(X, Y) \geq 0$$

$$I(X; Y) \geq 0$$

DIAGRAM:



$$H(X, Y) = H(X|Y) + \cancel{I(X; Y)} + H(Y|X).$$

✓ $H(Y) - H(Y|X)$

$$\frac{H(X|Y) + H(Y) - \cancel{H(Y|X)}}{H(X, Y)} + \cancel{H(Y|X)}$$

$$= H(X, Y).$$